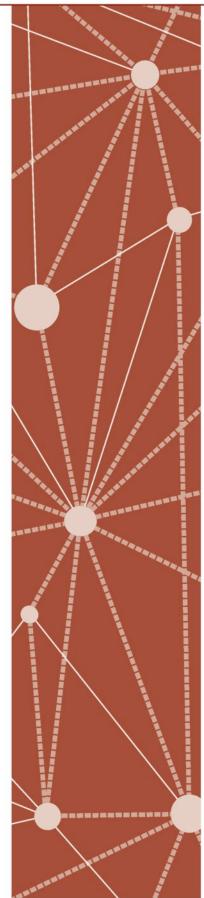
CONCEPT DEVELOPMENT



Mathematics Assessment Project CLASSROOM CHALLENGES A Formative Assessment Lesson

Mean, Median, Mode, and Range

Mathematics Assessment Resource Service University of Nottingham & UC Berkeley Beta Version

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Mean, Median, Mode, and Range

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Calculate the mean, median, mode, and range from a frequency chart.
- Use a frequency chart to describe a possible data set, given information on the mean, median, mode, and range.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

6-SP: Develop understanding of statistical variability. Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 4. Model with mathematics.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task designed to reveal their current understanding and difficulties. You then review their responses and create questions for students to consider when improving their work.
- After a whole-class introduction, students work in small groups on a collaborative task, matching bar charts with statistical tables.
- To end the lesson there is a whole-class discussion.
- In a follow-up lesson, students again work alone on a task similar to the initial assessment task.

MATERIALS REQUIRED

- Each student will need a copy of the two assessment tasks: *Penalty Shoot-Out* and *Boy Bands*, a mini-whiteboard, a pen, and an eraser.
- Each small group of students will need *Card Set: Bar Charts, Card Set: Statistics Tables* (already cut-up), a large sheet of paper, and a glue stick.
- There is a projector resource to support whole-class discussions.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 90-minute lesson, and 15 minutes in a followup lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: Penalty Shoot-Out (15 minutes)

Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task: *Penalty Shoot-Out*.

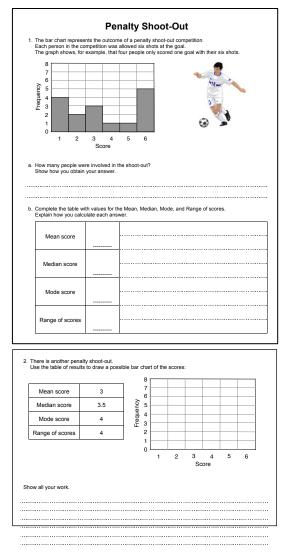
Read through the questions and try to answer them as carefully as you can.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot understand or do everything, because in the next lesson they will work on a similar task, which should help them. Explain to students that by the end of the next lesson, they should be able to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.



We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students.

Common issues	Suggested questions and prompts
Misinterprets the axes on the bar chart For example: The student states that there were six people involved in the shoot-out (Q1a). Or: The student does not understand the term 'Frequency'.	 Complete this sentence "This bar shows that" (indicate one of the bars). What does the term 'Frequency' mean? How many people scored three goals? How many people scored four goals?
Uses incorrect values when calculating the mean For example: The student finds the total of the frequencies rather than the total number of goals. Or: The student divides by six rather than the total frequency. Or: The student adds the scores: (1 + 2 + 3 + 4 + 5 + 6) and divides this total by 6.	 How many goals were scored? Six goals were scored five times. So what is the total number of goals? Compare this to your total, what do you notice? Imagine writing the scores out as a list. From this list, how would you work out the mean?
Confuses the position of the median with the value for the median For example: The student adds one to the total frequency and divides by two to give a median of 8.5 (Q1b). Or: The student just divides the frequency by two (Q1b). Or: The student assumes the median is 3.5, half way between 1 and 6 Or: The student writes two values for the median, 3 and 4.	 The median is the middle score when all the scores are in order. Is this what you have found? Try writing the scores in order: 1,1,1,1,2,2,3, Which is the middle score? How could you do this directly from the frequency graph without writing a list?
Presents the range as two figures, the highest and the lowest scores	• What calculation is needed to obtain the range?
Calculates the range in frequencies rather than the range of goals scored.	What was the highest number of goals scored?What was the lowest number of goals scored?
Reads off the frequency of the tallest bar as the mode, rather than the score For example: The student gives the mode as 5 Q1b.	• Which score was the most popular? How can you tell?
Draws a bar chart that satisfies none or some of the criteria given in the table (Q2) For example: The student draws a bar chart with a mode of 4 but the other values in the table are not satisfied. Completes the task	 Check that your bar chart works for all the values in the table. What is the mean/median/mode/range? Can you use the bar chart to draw a frequency table? Can you produce a different bar chart (to Q2)
The student needs an extension task	that describes the same data measures? What is the same and what is different?

SUGGESTED LESSON OUTLINE

Whole-class introduction (20 minutes)

Give each student a mini-whiteboard, a pen, and an eraser.

Display Slide P-1 of the projector resource:



Many students may be aware of rating systems used on popular websites. Ask students to name a computer game that most people know. If more than one computer game is suggested then you may want to ask the class to vote on which one they want to rate.

Once the computer game has been agreed upon, ask students to rate the game by writing a score between 1 and 6 on their whiteboards (if you prefer, you could use pieces of paper or card rather than whiteboards.)

How would you rate the game on a scale of 1 to 6 where 1 = poor and 6 = great? On your whiteboard [paper] show me your score for the game. It must be whole number e.g. $2\frac{1}{2}$ is not allowed.

The results of the student survey will be used to produce a bar chart from which the process of using the bar chart to find the mean, median, mode, and range will be discussed.

Before you do this, question students on efficient ways of recording the data collected in the class. The focus here is on an efficient method for collecting the scores rather than different ways of displaying the data.

You have each got a score for the game. How can we record the scores for the class on the board?

Students may suggest writing a list of the responses or creating a tally. Discuss the benefits of using a list or tally when the data is not being collected simultaneously e.g. surveying makes of cars driving past a certain point. Emphasize the difference between this kind of data collection and the data that has just been collected by the class, whilst highlighting the importance of using an efficient method.

If students have not already suggested it, introduce the idea of a frequency table and check that students understand the term 'Frequency':

In math, what does the word 'Frequency' mean? In this case, can you think of an equivalent phrase? Why do we use 'Frequency' instead of (the equivalent phrase)? [Frequency is a general term that can be used when working with data. It is usually an abbreviation of a longer, more specific phrase.] Display Slide P-2 of the projector resource:

7 6 6 7 6 7 6 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 7 6 7	Bar Chart fr	om	al	re	que	enc	УT	able
8 Mean score 5 Median score 3 Mode score 2 Panne of scores	Score	1	2	3	4	5	6]
6 Mean score 5 Median score 4 Median score 3 Mode score 2 Panne of scores	Frequency							1
1 Nullge of scores	8							

Use the scores on the students' whiteboards to complete the frequency table, then ask a volunteer to come out and complete the bars on the bar chart.

We are now going to find the mean, median, mode, and range of scores for the game.

Check that students understand the meaning of the terms 'mean', 'median', 'mode', and 'range'. Ask students to come out and demonstrate the four calculations using the information collected and notice whether they choose to use the frequency table or the bar chart. Emphasize using the bar chart directly as this is what the students will be doing when completing the collaborative task:

In this lesson you will be matching information displayed in a bar chart with values for the mean, median, mode and range.

Without writing anything down, how can we calculate the median, mode, and range from the bar chart?

Students may struggle to find the median directly from the bar chart and may prefer to write the data points in order and cross out until the 'middle' number is reached. If this is the case, spend some time exploring different strategies for finding the 'median' directly from the bar chart. Depending on the data collected, it may also be appropriate to discuss the method of finding the median score where there is no 'middle number'.

Hold a discussion on calculating the mean:

How can we calculate the mean score?

It is likely that students will need to write some values down when finding the mean. Spend some time discussing possible ways of calculating the mean score using the bar chart, without writing out a list of the raw scores, for example by multiplying frequencies by scores then summing.

Collaborative small-group work: *Matching Cards* (35 minutes)

Organize the class into groups of two or three students and give each group *Card Set: Bar Charts* and *Card Set: Statistics Tables* (already cut-up), a large sheet of paper and a glue stick.

Explain how students are to work collaboratively. Slide P-3 of the projector resource summarizes these instructions.

Take turns to match a bar chart with a statistics table. Place the cards side by side on your desk, rather than on top of one another, so that everyone can see them.

Each time you match a pair of cards explain your thinking clearly and carefully. You may want to use your mini-whiteboards for any calculations and/or when explaining to each other what you have done.

Partners should either agree with the explanation, or challenge it if it is unclear or incomplete.

Once agreed, stick the cards onto the poster paper writing any relevant calculations and explanations next to the cards.

You will notice that some of the statistics tables have gaps in them and one of the bar charts cards is blank. Try to work out what these blanks should be and complete the cards before finalizing your matches.

While students are working in small groups you have two tasks: to note different student approaches to the task and to support students working as a group.

Note different student approaches

Listen and watch students carefully. In particular, notice how students make a start on the task, where they get stuck, and how they overcome any difficulties.

Do students start with a bar chart and calculate the mean, median, mode and range and then see if there is a statistics table that matches? If so, which average do they calculate first? Do they check to see if there are any other statistics tables that might also work? Do they sort the statistics tables? If so, how? Do they use a process of elimination? If so, what measure do they use to do this? How do they use the statistics table to complete the blank bar chart card? When finding the range from a graph with a frequency of zero for either a score of 1 or 6, do they still include these scores for the maximum or minimum values?

Support students working as a group

As students work on the task support them in working together. Encourage them to take turns and if you notice that only one partner is matching cards, ask other students in the group to explain the match.

Carl matched these two cards. Jess, why does Carl think these two cards match?

If students in the group take different approaches when matching cards, encourage them to clearly explain the basis for a choice. Try to avoid giving students the information they need to match pairs. Instead, encourage students to interpret the cards by careful questioning, for example:

Which two bar chart cards show a sample size of 12? How do you know? Can we tell how big the sample size is from the statistics table? Why/Why not? Is there more than one way of completing the blank bar chart? What is the sample size for the bar chart you have drawn? Can you draw a bar chart with a different sample size that still satisfies the values in the statistics table?

Some groups may ignore the blanks on the cards. Check that they are filling these in as they complete their matches. It is not essential that students complete all of the matches but rather that they are able to develop effective strategies for matching the cards and can justify their matches.

You may want to draw on the questions in the *Common issues* table to support your own questioning.

Whole-class discussion: sharing strategies

During the collaborative small-group work you may want to hold a brief class discussion about the strategies being used within the class. This may help students who are struggling to get started.

Jane what have you done so far? Can you explain your reasons for your chosen strategy? Has anyone else used a different strategy?

Rather than promoting a particular strategy, focus the discussion on exploring different possible methods of working:

Do you need to calculate all four statistics in turn for each bar chart? If not, why not?

Which statistic is easiest to find? How might you use this to complete the task? What similarities or differences could you look for in the cards? How could you use what you notice?

If students are still struggling with the task, suggest that they focus on the cards with all four statistics values completed (S1, S4, S5 & S7) and see if they can find bar charts to match these four cards. Alternatively they may find it helpful to group the cards in some way e.g. all cards with a range of 3.

Sharing posters (15 minutes)

Once students have finished their posters, ask them to share their work by visiting another group. This gives the students the opportunity to engage at a deeper level with the mathematics and encourages a closer analysis of the work than may be possible by students presenting their posters to the whole class. It may be helpful for students to jot down the pairs of cards matched on their mini-whiteboards, for example, B2 and S6 etc. before they visit another group.

Now, **one** person from each group get up and visit a different group and look carefully at their matched cards.

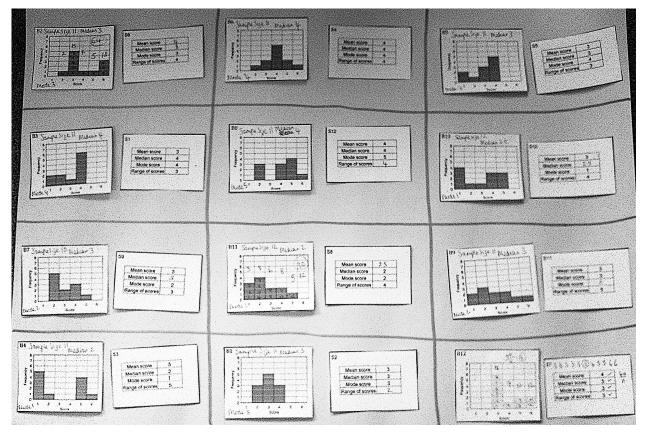
Check the cards and point out any cards you think are incorrect.

You must give a reason why you think the card is incorrectly matched or completed, but do not make changes to the card.

Once students have checked another group's cards, they may need to review their own cards, taking into account comments from their peers. They can make any necessary changes by drawing arrows to where a particular bar chart or statistics table should have been placed.

Slide P-4 of the projector resource summarizes these instructions.

The finished poster may look something like this:



Whole-class discussion (20 minutes)

It is likely that some groups will not have matched all of the cards, but the aim of this discussion is not to check answers but to explore the different strategies used by students when matching/completing the cards, as well as identifying areas in which students struggled.

First select a pair of cards that most groups correctly matched. This approach may encourage good explanations. Then select one or two cards that most groups found difficult to match.

Once one group has justified their choice for a particular match, ask other students to contribute ideas for alternative strategies, and their views on which reasoning method was easier to follow. The intention is that you focus on getting students to understand and share their **reasoning**.

Use your knowledge of the students' individual and group work to call on a wide range of students for contributions.

Which cards were the easiest to match? Why was this? Which cards were difficult to match? Why was this? When matching the cards, did you always start with the bar chart/statistics table? Why was this? Did anyone use a different strategy?

You may again want to draw on the questions in the *Common issues* table to support your own questioning.

Follow-up lesson: Reviewing the assessment task (15 minutes)

Give each student a copy of the assessment task *Boy Bands*, and their original scripts from the assessment task *Penalty Shoot-Out*. If you have not added questions to individual pieces of work, then write your list of questions on the board. Students select from this list only those questions they think are appropriate to their own work.

Read through your papers from Penalty Shoot-Out and the questions [on the board/written on your script.] Think about what you have learned.

Now look at the new task sheet, Boy Bands. Can you use what you have learned to answer these questions?

If students struggled with the original assessment task, you may feel it more appropriate for them to revisit *Penalty Shoot-Out* rather than attempt *Boy Bands*. If this is the case give them another copy of the original assessment task instead.

SOLUTIONS

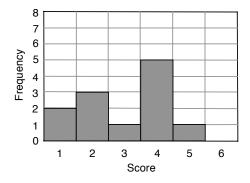
Assessment task: Penalty Shoot-Out

1a. There are 16 people involved in the shoot-out. This is the sum of all the frequencies on the bar chart.

b.

Mean score	3.5
Median score	3
Mode score	6
Range of scores	5

2. A possible solution is:



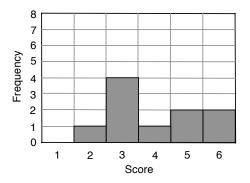
Assessment task: Boy Bands

1a. There are 12 people participating in the quiz.

b.

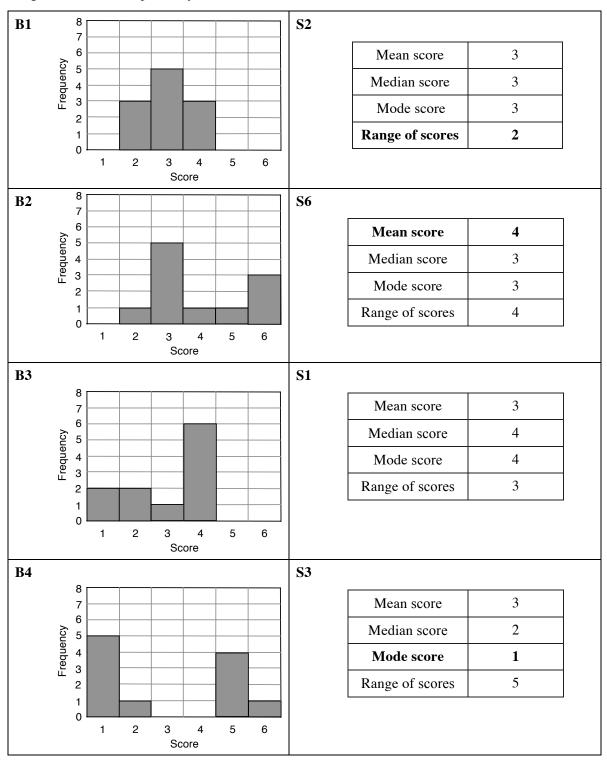
Mean score	3
Median score	2.5
Mode score	5
Range of scores	4

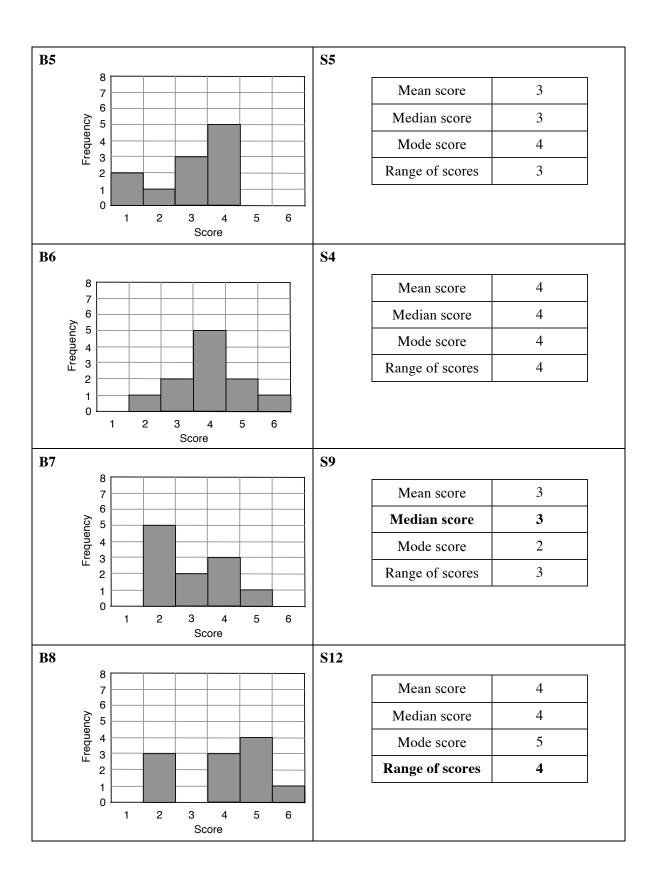
2. A possible solution is:

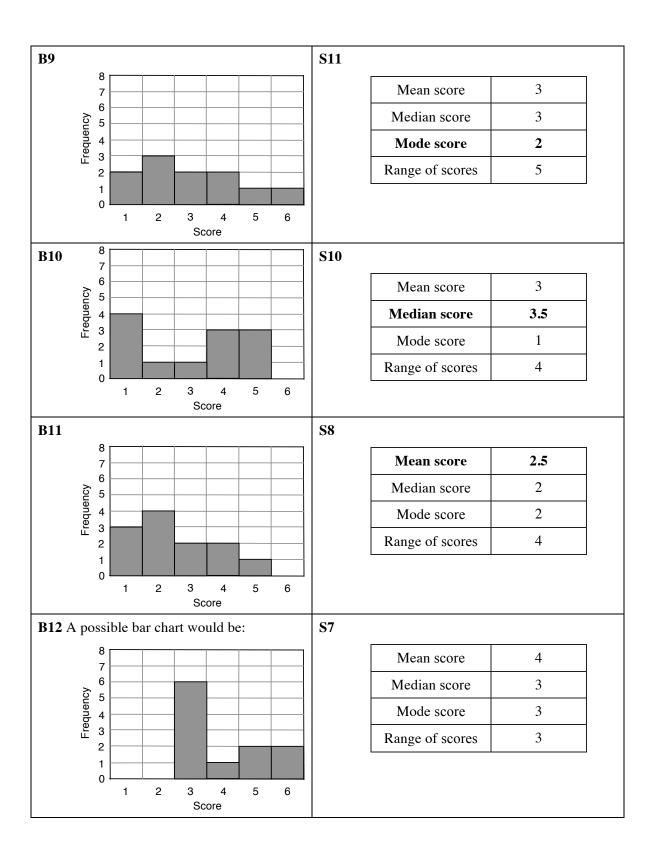


Collaborative Activity: Card Matching

Missing values to be completed by students are in **bold**.

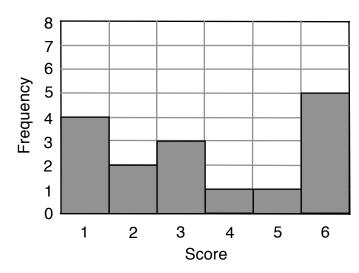






Penalty Shoot-Out

 The bar chart represents the outcome of a penalty shoot-out competition. Each person in the competition was allowed six shots at the goal. The graph shows, for example, that four people only scored one goal with their six shots.





a. How many people were involved in the shoot-out? Show how you obtain your answer.

Complete the table with values for the Mean Median Media and Dense of second

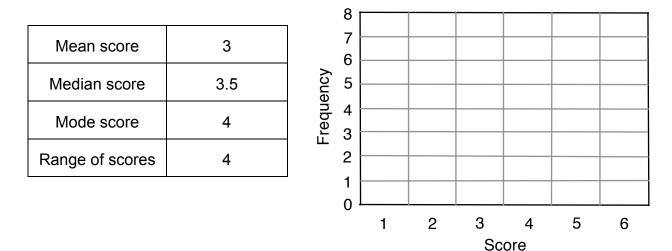
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b. Complete the table with values for the Mean, Median, Mode, and Range of scores. Explain how you calculate each answer.

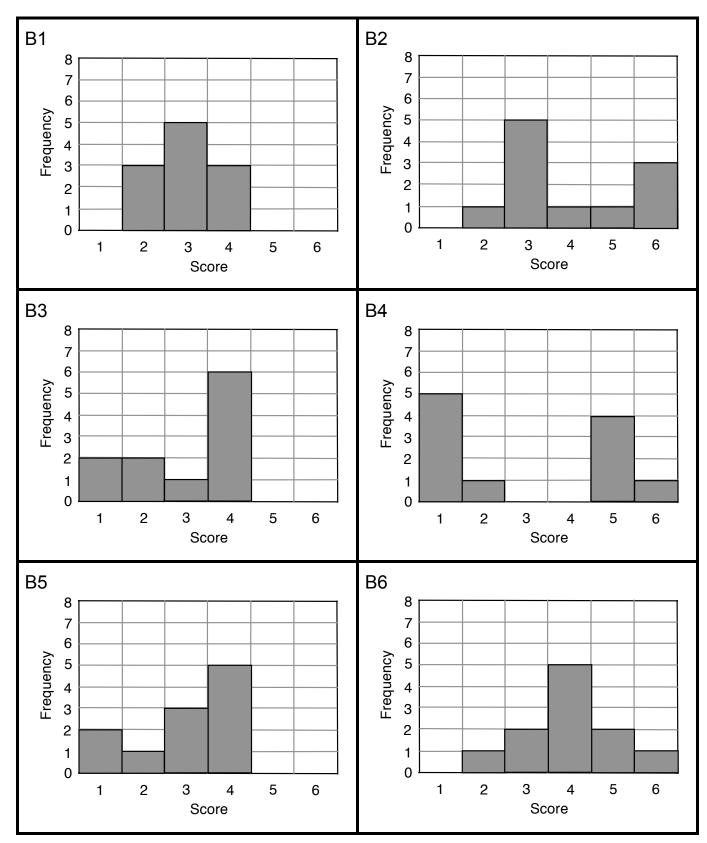
Mean score	
Median score	
Mode score	
Range of scores	

2. There is another penalty shoot-out.

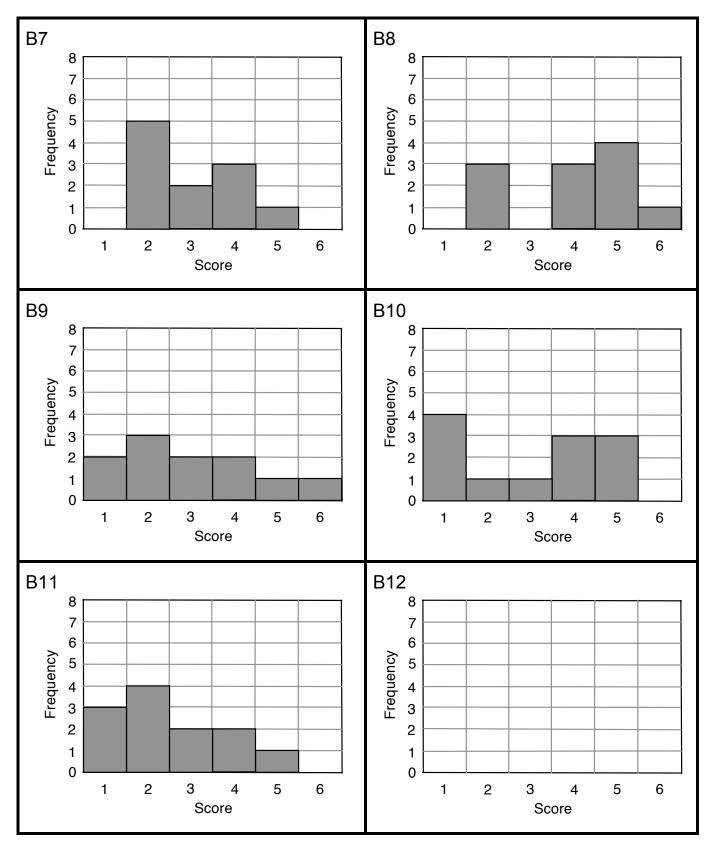
Use the table of results to draw a possible bar chart of the scores:



Show all your work.



Card Set: Bar Charts



Card Set: Bar Charts (continued)

Student Materials

Card Set: Statistics Tables

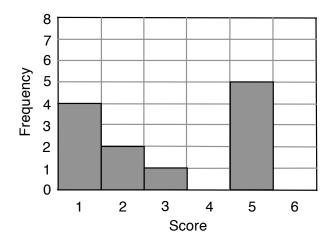
S1			S	2	
Mean s	score	3		Mean score	3
Median	score	4		Median score	3
Mode s	score	4		Mode score	3
Range of	scores	3		Range of scores	
S3			S	4	
Mean s	core	3		Mean score	4
Median	score	2		Median score	4
Mode s	core			Mode score	4
Range of	scores	5		Range of scores	4
S5			S	6	
Mean s	core	3		Mean score	
NA a alla sa	score	3		Median score	3
Median		4		Mode score	3
Median Mode s	core				

Card Set: Statistics Tables (continued)

S7			S8		
	Mean score	4		Mean score	
	Median score	3		Median score	2
	Mode score	3		Mode score	2
	Range of scores	3		Range of scores	4
S9			S10		
	Mean score	3		Mean score	3
	Median score			Median score	
	Mode score	2		Mode score	1
	Range of scores	3		Range of scores	4
S1 ⁻	1		 S12		
	Mean score	3		Mean score	4
		3		Median score	4
	Median score	-			
	Mode score			Mode score	5

Boy Bands

 The bar chart represents the scores from a quiz. Children were asked to name six boy bands in 30 seconds. Each score represents the number of correctly named bands.





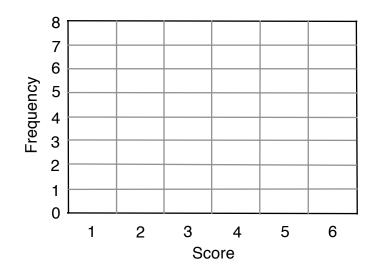
a. How many children were involved in the quiz? Show how you obtain your answer.

b. Complete the table with values for the Mean, Median, Mode, and Range of scores. Explain how you calculate each answer.

Mean score	
Median score	
Mode score	
Range of scores	

2. The results of another quiz question is shown in the table below. Draw a possible bar chart of the scores:

Mean score	4
Median score	3.5
Mode score	3
Range of scores	4



Show all your work.

Computer Games: Ratings

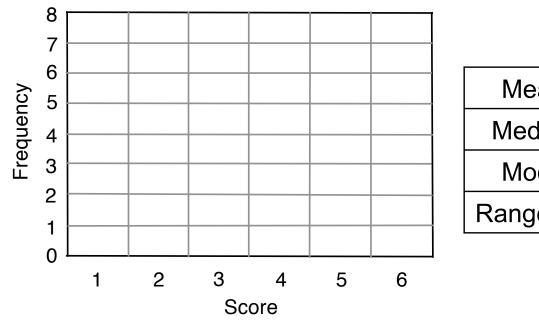


Imagine rating a popular computer game.

You can give the game a score of between 1 and 6.

Bar Chart from a Frequency Table

Score	1	2	3	4	5	6
Frequency						



Mean score	
Median score	
Mode score	
Range of scores	

Matching Cards

- 1. Each time you match a pair of cards, explain your thinking clearly and carefully.
- 2. Partners should either agree with the explanation or challenge it if it is unclear or incomplete.
- 3. Once agreed stick the cards onto the poster and write a justification next to the cards.
- 4. Some of the statistics tables have gaps in them and one of the bar charts is blank. You will need to complete these cards.

Sharing Posters

- 1. One person from each group visit a different group and look carefully at their matched cards.
- 2. Check the cards and point out any cards you think are incorrect. You must give a reason why you think the card is incorrectly matched or completed, but do not make changes to the card.
- 3. Return to your original group, review your own matches and make any necessary changes using arrows to show if card needs to move.

Mathematics Assessment Project CLASSROOM CHALLENGES

This lesson was designed and developed by the Shell Center Team at the University of Nottingham

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with

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It was refined on the basis of reports from teams of observers led by David Foster, Mary Bouck, and Diane Schaefer

based on their observation of trials in US classrooms along with comments from teachers and other users.

This project was conceived and directed for MARS: Mathematics Assessment Resource Service

by

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and based at the University of California, Berkeley

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Illustrative Mathematics

7.SP.3,4 - College Athletes

Alignment 1: 7.SP.B.3, 7.SP.B.4

Field Hockey

Basketball

63

Not yet tagged

Below are the heights of the players on the University of Maryland women's basketball team for the 2012-2013 season and the heights of the players on the women's field hockey team for the 2012 season. (Accessed at http://www.umterps.com/sports/w-fieldh/mtt/md-w-fieldh-mtt.html , http://www.umterps.com/sports/w-baskbl/mtt/md-w-baskbl-mtt.html on 1/13/13) Note: it is typical for a women's field hockey team to have more players than a women's basketball team would.

Field Hockey Player Heights (inches)	Basketball Player Heights (inches)
66	75
64	65
66	76
63	75
67	76
62	72
62	67
64	69
64	74
64	68
65	74
66	79
65	
64	1
63	1
62	1
62	1
68	1
68	
66]
70]
67	
65]
62]
64	1

a. Based on visual inspection of the dotplots, which group appears to have the larger average height? Which group appears to have the greater variability in the heights?

69

Heights (in inches)

66

72

75

78

- b. Compute the mean and mean absolute deviation (MAD) for each group. Do these values support your answers in part (a)?
- c. How many of the 12 basketball players are shorter than the tallest field hockey player?
- d. Imagine that an athlete from one of the two teams told you she needs to go to practice. You estimate that she is about 65 inches tall. If you had to pick, would you think that she was a field hockey player or that she was a basketball player? Explain your reasoning.
- e. The women on the Maryland field hockey team are not a random sample of all female college field hockey players. Similarly, the women on the Maryland basketball team are not a random sample of all female college basketball players. However, for purposes of this task, suppose that these two groups can be regarded as random samples of all female college field hockey players and all female college basketball players, respectiviely. If these were random samples, would you think that female college basketball players are typically taller than female college field hockey players? Explain your decision using answers to the previous questions and/or additional analysis.

Commentary

In this task, students are able to conjecture about the differences in the two groups from a strictly visual perspective and then support their comparisons with appropriate measures of center and variability. This will reinforce that much can be gleaned simply from visual comparison of appropraite graphs, particularly those of similar scale. Students are also encouraged to consider how certain measurements and observation values from one group compare in the context of the other group. As a possible extension, students can investigate if these distributions are in fact similar to the distributions of heights of women's field hockey and women's basketball players.

Task 1341 is similar to this task and looks at weights of two groups of offensive linemen. In that task, the difference in MAD's is also about 2. However, the variability for the two groups was judged to be similar because in the context of that task, a difference of 2 pounds is small realtive to the weight values, which ranged from 250 to 340 pounds. In this task, a difference of 2 inches is judged as meaningful relative to the height values in the data sets.

Solution: Solution

- a. The center of the basketball distribution is much higher on the number line than the center of the field hockey distribution, so at first glance, it appears that the basketball group has the higher average. Similarly, the values for the basketball distribution appear to have a greater range and are less concentrated than the field hockey distribution, so it appears that the basketball group has greater variability in its observations.
- b. Field Hockey: mean = 64.76, MAD = 1.75; Basketball: mean = 72.5, MAD = 3.58. These values do support the conjectures from Part (a).
- c. The tallest field hockey player is 70 inches. Four of the basketball players are less than 70 inches (65, 67, 68, and 69).
- d. At 65 inches, she is more likely to be a field hockey player. Using the summary measures, 65 inches is approximately the mean for the field hockey players, so she would be a field hockey player of average height. A height of 65 inches is more unusual for the basketball team as that value is just over 2 MAD's below the mean. Using the raw data and a probability argument, 3 of the 25 field hockey players are 65 inches (12%) and only one out of the 12 basketball players is 65 inches (8.3%)
- e. Yes, it appears that women's college basketball players are typically taller than women's college field hockey players. In addition to any arguments/statements made earlier regarding the dotplots and summary measures, one could also mention that $\frac{2}{9}$ of the basket players are taller than the tallest field hockey player (and similar comparative arguments)



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Illustrative Mathematics

8-SP.4 What's Your Favorite Subject?

Alignment 1: 8.SP.A.4

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Favorite Subject by Grade					
Grade	English	History	Math/Science	Other	Totals
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate relative frequencies using the given data.

Commentary:

Either row percentages or column percentages are appropriate for the solution, since there is no clear explanatory/response relationship between the variables. Whether the student sees a strong association or not is less important than whether his or her answer uses the data appropriately and understands that an association means that the distribution of favorite subject is different for 7th graders and 8th graders.

Another approach is to observe that 39% of the students are 7th graders. If grade is not associated with favorite subject, you would expect about 39% of the students picking a particular subject as their favorite to be 7th graders.

Solution: Possible Solution

Row relative frequencies (that is, distribution of the variable "Favorite Subject" for each grade) are given in the table below.

Favorite Subject by Grade				
Grade	English	History	Math/Science	Other
7th Grade	0.328	0.310	0.241	0.121
8th Grade	0.258	0.247	0.396	0.099

There is some association between favorite subject and grade. A higher percentage of 7th graders than 8th graders prefer English and History (and - to a lesser extent - "other" subjects), while a higher percentage of 8th graders than 7th graders prefer Math/Science.



 OOO
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COUNTING TREES

This diagram shows some trees in a tree farm.

The circles \bullet show old trees and the triangles \blacktriangle show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

- 1. What method could he use to estimate the number of trees of each type? Explain your method fully.
- 2. On your worksheet, use your method to estimate the number of:
 - (a) Old trees
 - (b) Young trees

COUNTING TREES WORKSHEET

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CANDY BARS



A group of friends are planning to sell candy bars at the school shop.

They conduct a small survey among 30 people, asking the question:

How many candy bars do you eat in a typical week?

Here are their results:

Male	Female	Male	Female	Male	Male
1 bar	4 bars	5 bars	1 bar	2 bars	25 bars
Male	Female	Male	Male	Male	Female
13 bars	0 bars	2 bars	9 bars	6 bars	16 bars
Female	Male	Male	Male	Female	Male
14 bars	10 bars	19 bars	11 bars	1 bar	0 bars
Male	Male	Female	Male	Female	Male
1 bar	3 bars	10 bars	25 bars	16 bars	13 bars
Female	Male	Male	Male	Male	Female
30 bars	8 bars	2 bars	0 bars	28 bars	0 bars

- 1. Draw graphs or charts to compare the results for males and females.
- 2. Chris says:
- "We have found that the total number of bars eaten by all the males is 183, and the total number eaten by all the females is 92. In general, this means that men eat more candy than women."
- (a) Give two reasons why Chris is wrong in his reasoning.
- (b) Write down **one** conclusion (comparing males and females) that is supported by the data. Show any work you do.

Illustrative Mathematics

8.SP.1 Hand span and height (has both reviews)

Alignment 1: 8.SP.A.1

Do taller people tend to have bigger hands? To investigate this question, each student in your class should measure his or her hand span (in cm) and height (in inches). Record these values in the table below.

Student	Hand Span (cm)	Height (inches)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

a. Create a clearly labeled graph that displays the relationship between height and hand span.

b. Based on the graph, how would you answer the question about whether taller people tend to have bigger hands?

c. Based on your graph, would you describe the relationship between hand span and height as linear or nonlinear? Explain your choice.

Commentary:

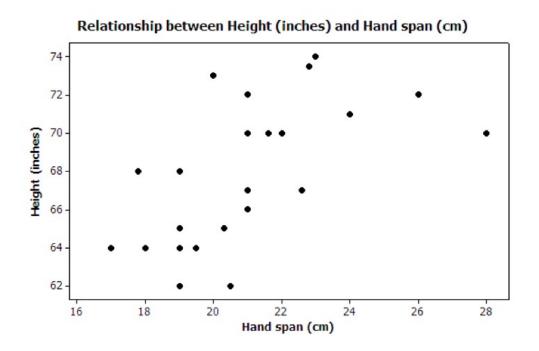
- For purposes of consistency, it is recommended that one person be in charge of taking the measurements. This role may be taken by the instructor, or one of the students in class.
- For consistency, measure hand span of right hand for all students.
- To measure hand span, spread out fingers as much as possible, and then measure the distance (cm) between tip of thumb to tip of little finger.
- Height should be measured without shoes and without anything on the head that would inflate height (besides hair). For those wearing religious headwear, be careful not to measure the height of the headwear.
- Notice that hand span is measured in centimeter which is a finer unit of measurement, to account for the fact that a difference in hand span is on a relatively finer scale compared to difference in height, which can be measured in inches (and is typically measured in feet and inches).
- Typically, the association between hand span and height has observed to be positive, moderately strong, and linear, with relatively
 few outliers. That is, people with larger hand spans tend to be taller. Also, depending, there may be some noticeable separation of
 males and females, with heights and hand spans of females being towards the left bottom corner of the scatterplot, and those for the
 males being towards the right top corner. Of course, there may be outliers in this case, too.

Solution: Solution

Solutions will vary depending on the actual data values collected by the class. Below is a solution based on the following hypothetical data set:

Student	Hand Span (cm)	Height (inches)
1	17.0	64.0
2	21.0	67.0
3	20.3	65.0
4	26.0	72.0
5	24.0	71.0
6	22.0	70.0
7	21.0	66.0
8	19.0	62.0
9	20.0	73.0
10	19.0	65.0
11	17.8	68.0
12	20.5	62.0
13	21.0	70.0
14	22.8	73.5
15	22.6	67.0
16	21.0	72.0
17	23.0	74.0
18	21.6	70.0
19	21.0	72.0
20	28.0	70.0
21	18.0	64.0
22	19.0	68.0
23	19.5	64.0
24	19.0	64.0

a. Here is a scatterplot showing the relationship between height and hand span.



- b. Each dot represents a student, and the position of the dot with regard to the horizontal axis represents the student's hand span (cm), whereas the position of the dot with regard to the vertical axis represents the student's height. We can see that dots with lower hand span values also tend to have lower values for height; similarly dots with higher hand span values also tend to have higher values for height. Overall, we can see that there is an upward trend in the scatterplot. This shows that taller people tend to have bigger hand spans.
- c. The overall form of the relationship between height and hand span appears to be linear, except for the student with a hand span of 28cm and height of 70inches. We can say this because a line seems to be the most appropriate pattern to represent how height is changing with hand span.

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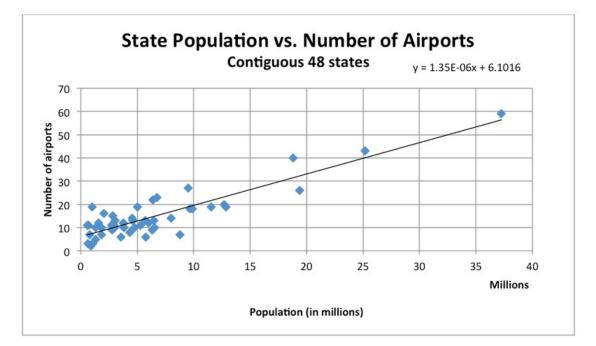
Illustrative Mathematics

8.SP US Airports

Alignment 1: 8.SP.A.3

Not yet tagged

The scatter plot below shows the relationship between the number of airports in a state and the population of that state according to the 2010 Census. Each dot represents a single state.



- a. How would you characterize the relationship between the number of airports in a state and the state's population? (Select one):
 - i. The variables are positively correlated; states with higher populations tend to have fewer airports.
 - ii. The variables are negatively correlated; states with higher populations tend to have fewer airports.
 - iii. The variables are positively correlated; states with higher populations tend to have more airports.
 - iv. The variables are negatively correlated; states with higher populations tend to have more airports.
 - v. The variables are not correlated.

LaToya uses the function $y = (1.35 \times 10^{-6})x + 6.1$ to model the relationship between the number of airports, y and the population in a state, x.

- b. How many airports does LaToya's model predict for a state with a population of 30 million people? [_____].
- c. What does the number 6.1 that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)
 - i. The average number of airports in a state is 6.1.
 - ii. The median number of airports in a state is 6.1.
 - iii. The model predicts a population of 6.1 people in a state with no airports.
 - iv. The model predicts 6.1 airports in a state with no people.
 - v. The model predicts that 6.1 states have no airports.
 - vi. The model predicts 6.1 more airports, on average, for each additional person in a state.
 - vii. The model predicts 6.1 fewer airports, on average, for each additional person in a state.

- viii. The number 6.1 cannot be interpreted in this context.
- d. What does the number 1.35×10^{-6} that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)
 - i. The average number of airports in a state is $1.35 imes 10^{-6}.$
 - ii. The median number of airports in a state is $1.35 imes 10^{-6}$.
 - iii. The model predicts $1.35 imes 10^{-6}$ airports in a state with no people.
 - iv. The model predicts $1.35 imes 10^{-6}$ people in a state with no airports.
 - v. The model predicts that 1.35×10^{-6} states have no airports.
 - vi. The model predicts $1.35 imes 10^{-6}$ more airports, on average, for each additional person in a state.
 - vii. The model predicts $1.35 imes 10^{-6}$ fewer airports, on average, for each additional person in a state.
 - viii. The number $1.35 imes 10^{-6}$ cannot be interpreted in this context.
- e. Fill in the following newspaper headline based on this relationship:

On average, a state in the contiguous 48 US states has 1 additional airport for every

Commentary

Purpose

This is one of two assessment tasks illustrating the similarities and differences between the 8th grade standards in Functions and in Statistics and Probability. The first, 8.F Mail Truck, involves a situation that can be modeled exactly with a linear function. The second, 8.SP US Airports, uses a linear function to model a relationship between two quantities that show statistical variation and do not have an exact linear relationship.

In 8.SP US Airports, each additional person in the state does not directly correspond to a portion of an airport, but the relationship can be modeled using a linear association, and the model can be used to make predictions about the number of airports in states with a given population. In 8.F Mail Truck, each additional day of driving does correspond to exactly the same increase in the number of miles put onto the truck each day.

Cognitive Complexity

Mathematical Content

This task involves constructing a linear function and interpreting its parameters in a context. Thus, this task has a medium level of complexity.

Mathematical Practice

The task asks students to reason abstractly and quantitatively (MP 2) and directly assesses component skills related to mathematical modeling (MP 4), namely, interpreting mathematical objects in contexts.

Linguistic Demand

This context in this task requires students to interpret the mathematics in this context, so has a high level of linguistic complexity.

Stimulus Material

The stimulus material is not complex.

Response Mode

The interface is not complex.

Solution: 1

a. (iii)

b. 46.6 airports

c. (iv)

d. (vi)

e. 700 thousand

This is a 4-point item.

3 points if a student misses one part.

2 points if a student misses 2 parts

1 point if a student misses 3 parts.

0 points if a student misses 4 or 5 parts.

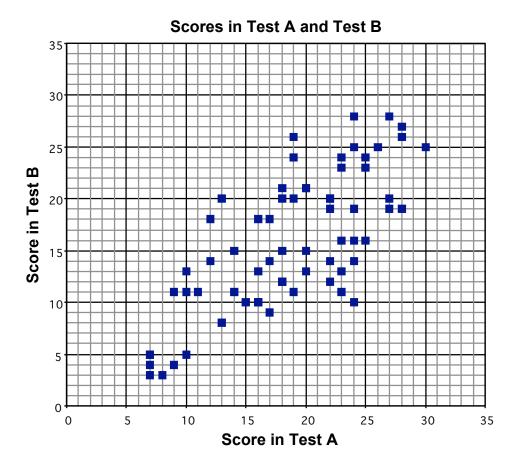


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Scatter Diagram

A group of 66 students took two tests; Test A and Test B.

In the scatter diagram, each square represents one student and shows the scores that student got in the two tests.



- The mean score for Test A was 19 and the mean score for Test B was 16.
 Plot a point to show this on the scatter diagram.
- Draw a line of best fit on the scatter diagram.
 How can a line of best fit be used?

3. Here are five statements about the scores shown on the scatter diagram.

If a statement is true check ($\sqrt{}$) it.

If it is not true, write a correct statement.

Statement	Check ($$) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	
The lowest score on Test A is lower than the lowest score for Test B.	
The range of scores on Test B is 25.	
The student with the highest score on Test A also has the highest score on Test B.	
The biggest difference between a student's scores on the two tests is 5.	